

# Comparator for Calibration of Inductive Voltage Dividers from 1 to 10 KHz<sup>\*</sup>

WILBUR C. SZE†

National Bureau of Standards  
Washington, D.C.

► A high-accuracy comparator is described for measuring the relative deviations in voltage ratios and phase angles of inductive voltage dividers. The new technique overcomes several inherent limitations of the existing comparison methods. The balance is accomplished by utilizing special shielded transformers and a resistance-capacitance network for in-phase and quadrature voltage injections. The measurements are accurate to within  $1 \times 10^{-7}$  of input in the frequency range from 1 to 10 KHz. Resolution is better than  $2 \times 10^{-10}$ .

## INTRODUCTION

THE INDUCTIVE voltage divider, since its first appearance about 15 yr ago, has proved to be an increasingly valuable tool in the field of accurate electrical measurements as its striking characteristics become more widely known.

The development of material of extremely high magnetic permeability has made possible the construction of inductive voltage dividers with excellent stability and with deviations in ratio and phase angle of less than  $5 \times 10^{-7}$  of input.<sup>(1)</sup> The accurate initial establishment of the voltage ratio and phase angle has been presented elsewhere and will not be repeated here.<sup>(2)</sup> These known voltage ratios and phase angles are preserved in the calibrated inductive voltage divider so that they may be used as a reference and reestablished again by a satisfactory comparison circuit without significant degradation in accuracy.

At the National Bureau of Standards, a new comparison method, based on suggestions by Robert D. Cutkosky, has been developed for relative ratio and phase angle measurements from 1 to 10 KHz to an accuracy of better than one part in  $10^7$ . This technique, which appears worthy of general use in standardizing laboratories, overcomes several inherent limitations of the

existing comparison methods: (1) the dividers are treated as four-terminal networks as they were designed to be operated; (2) the test circuit imposes a negligible burden on either the reference or the test divider; and (3) it is not necessary to use lower dials of either divider for in-phase voltage balancing.<sup>(3)</sup>

## DESCRIPTION OF CIRCUIT

This technique uses a double toroidal transformer, one transformer for null detection and the other for injection of differential voltages. Figure 1a shows the complete schematic circuit diagram simplified by omission of the shielding arrangements.  $D_1$  and  $D_2$  represent the reference and test dividers whose voltage ratios and phase angles are being compared. Their input and output terminals are connected together with short coaxial leads. The lead impedance between the input terminal of each divider and its junction point with the supply voltage should be very small and nearly equal. The detector circuit consists of a tuned null indicator and a specially constructed shielded transformer,  $T_3$ , to match the impedances and to isolate the detector from the measuring circuit. The parallel pair of inductive voltage dividers,  $D_3$  and  $D_4$ , inject through the R-C network (shown in Figure 1b) and transformer,  $T_2$ , in-phase and quadrature balance voltages into the lead connecting the output terminals of the dividers,  $D_1$  and

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<sup>†</sup>Electrical Engineer, High Voltage Laboratory; ISA Senior Member.

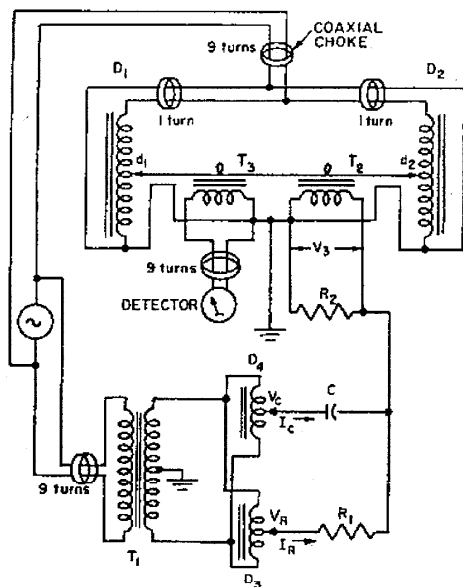


Figure 1a. Schematic circuit diagram.

$D_2$ . Transformer  $T_1$  is also specially constructed with an accurate 1/1 voltage ratio. The midpoint of the secondary winding is grounded so that both polarities of the injection voltages can be obtained from the outputs of  $D_3$  and  $D_4$  without the necessity of altering the circuit.

In order to realize an accuracy of better than one part in  $10^7$ , coaxial chokes are required in the circuit to reduce the small but troublesome loop and ground currents to a minimum.<sup>(4,5)</sup> Each coaxial choke consists of an appropriate number of turns of a coaxial cable threaded through a high-permeability core as indicated in Figure 1a.

Figures 2, 3, and 4 show the comparator and the circuit arrangement.

### THEORETICAL RELATIONS

By definition, the voltage ratio of a divider is the ratio of the secondary or output terminal voltage,  $V_{out}$ , to the primary or input terminal voltage,  $V_{in}$ , which can be expressed as

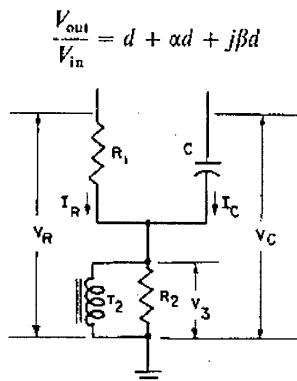


Figure 1b. R-C network.

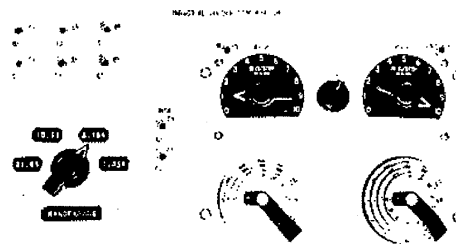


Figure 2. External view of comparator.

where  $d$  is the nominal ratio or setting of the dials. The term  $\alpha d$  is the in-phase deviation, and  $\beta d$  is the quadrature deviation. Both are expressed in parts per million (ppm) of input.

The equations for the relative in-phase deviation and quadrature deviation measurements can be derived from Figure 1b as follows:

$$V_R = I_R R_1 + V_3$$

$$V_C = -I_C X_C + V_3$$

$$V_3 = (I_C + I_R) \frac{R_2 Z}{R_2 + Z} \approx (I_C + I_R) R_2$$

where  $Z$  is the impedance of transformer  $T_2$ . Since in the circuit described here  $Z \gg R_2$ , the above approximation is justified.  $V_C$  and  $V_R$  are the voltages between the output terminals of  $D_3$  and  $D_4$  and ground.

By solving for  $I_R$  and  $I_C$  from the first two equations and substituting in the third,

$$V_3 = \frac{V_C R_1^2 + V_R X_C^2 (1 + R_1/R_2) + j X_C R_1 [V_C (1 + R_1/R_2) - V_R]}{R_1^2 + X_C^2 (1 + R_1/R_2)^2}$$

If  $r$  represents the turns ratio of  $T_2$ , then the voltage injected between output terminals of  $D_1$  and  $D_2$  is equal to  $V_3/r$ . However,  $V_C$  and  $V_R$  also can be expressed as decimal fractions of the supply voltage or by dial

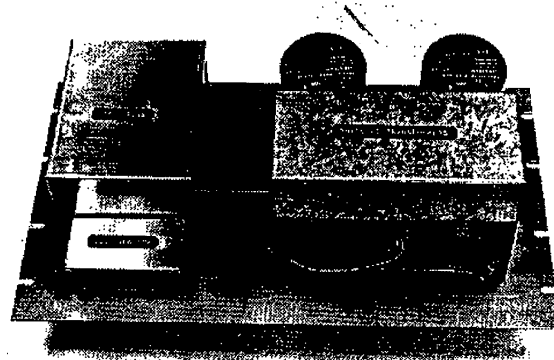


Figure 3. Internal view of comparator.

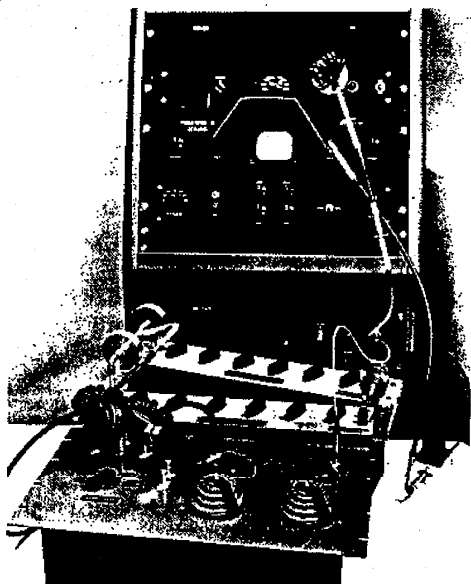


Figure 4. Circuit arrangement.

settings of  $D_3$  and  $D_4$  as  $d_C$  and  $d_R$ , respectively, and  $V_3$  as  $d_3$ .

Again from Figure 1a, when the detector indicates a null,

$$d_2(1 + \alpha_2 + j\beta_2) - d_1(1 + \alpha_1 + j\beta_1) = \frac{d_3}{r}$$

where  $d_1$  and  $d_2$  are the equal dial settings on the dividers  $D_1$  and  $D_2$ , at the point being compared. The terms  $\alpha$  and  $\beta$  are expressed in parts per million of dial setting  $d$ . By substituting the expression for  $d_3$  in the previous equation and separating real and  $j$  terms,

$$\alpha_2 - \alpha_1 = \frac{d_C R_1^2 + d_R X_C^2 (1 + R_1/R_2)}{rd_1 [R_1^2 + X_C^2 (1 + R_1/R_2)^2]}$$

$$\beta_2 - \beta_1 = \frac{R_1 X_C [d_C (1 + R_1/R_2) - d_R]}{rd_1 [R_1^2 + X_C^2 (1 + R_1/R_2)^2]}$$

If  $X_C \approx R_1$  and  $R_1 \gg R_2$ , then the equations for the relative deviations can be approximated as

$$d_1(\alpha_2 - \alpha_1) \approx \frac{d_R}{r(R_1/R_2)}$$

$$d_1(\beta_2 - \beta_1) \approx \frac{d_C R_2}{r X_C}$$

#### DESCRIPTION OF COMPONENTS

$T_1$ ,  $T_2$ , and  $T_3$  are specially constructed shielded transformers using high magnetic permeability toroidal cores to ensure close magnetic coupling and to conserve space in the circuit. The shields minimize the magnetic pickup from stray fields in the laboratory.

Figure 5 shows the schematic circuit of  $T_1$ . It consists of a toroidal core, an inner primary winding, an electro-

static shield, an outer secondary winding, and an overall Mu-metal case. Each winding is wound on the core in a single, equally distributed layer from  $0^\circ$  to  $360^\circ$ , and the wire is brought from  $360^\circ$  back to  $0^\circ$  or the point of origin. Thus, no loop is formed, and susceptibility to stray magnetic pickup is greatly reduced. (This same technique is used for winding  $T_2$  and  $T_3$ .) The midpoint of the secondary winding is grounded. The size of the core and number of turns for 1 to 5 KHz and 5 to 10 KHz operations at 100-V input are also indicated in Figure 5.

Figure 6 shows the details of construction of  $T_2$  and  $T_3$  transformers as a single unit.  $T_2$  and  $T_3$  are identical and interchangeable. Each consists of a toroidal core and a 100-turn primary winding. The shielding arrangements are such that the cross capacitances at the gaps are reduced to a minimum.<sup>(6)</sup> The secondary winding consists of a single conductor connected between the outputs of the test dividers. The capacitance between this conductor and the shield is less than 4 pF. Therefore, it imposes no significant burden on either divider.

The inductive voltage dividers,  $D_3$  and  $D_4$ , are commercially available units. Each consists of three decades and a continuously adjustable resistive divider. Their small voltage ratio and phase-angle deviations were determined and found to be negligible, because the injection voltage,  $V_3$ , rarely exceeds 0.002% of the input voltage.

$C$  is a three-terminal capacitor having a very small dissipation factor.  $R_1$  and  $R_2$  are three-terminal decade resistors of very small residual reactance. For 1-KHz calibrations,  $C$  is generally set to equal 15.92 nF,  $R_1 = 10$  K, and  $R_2 = 10 \Omega$ . (For 10-KHz calibrations, air capacitors of 1592 pF are used.) Thus, one step on the top decade of  $D_3$  and  $D_4$  represents a difference of 1 ppm in in-phase or quadrature deviation.

The detector circuit consists of a preamplifier and a cathode-ray oscillograph. A narrow band-pass filter is

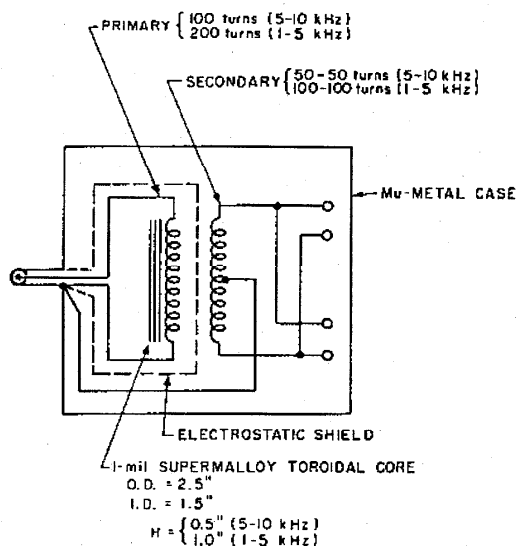


Figure 5. Shielded transformer.

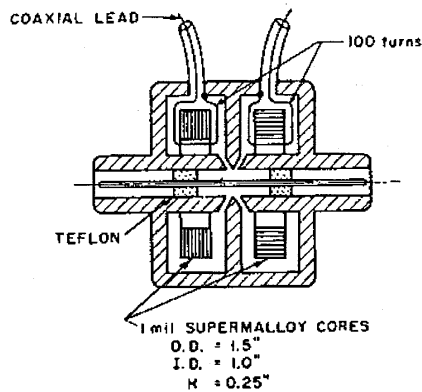


Figure 6. Injector-detector transformers.

used to discriminate against any noise before the first stage of amplification. A double elliptical pattern, suggested by Clothier, is shown on the cathode-ray oscillograph to indicate balance condition. The unbalance voltage from the comparison circuit is impressed on the vertical plates. On the horizontal plates is a signal of the same frequency whose phase can be adjusted so that angular departure of the major axis of the

elliptical pattern represents in-phase unbalance and opening of the minor axis quadrature unbalance. At balance the figure appears to be a single line.<sup>(2,3,7)</sup> The resolution is such that an unbalance voltage of 0.1 nV is easily observed.

## RESULTS

Table I lists the in-phase deviations,  $d_1\alpha_1$  and  $d_2\alpha_2$ , and quadrature deviations,  $d_1\beta_1$  and  $d_2\beta_2$ , of two inductive voltage dividers at 10 KHz and with an input of 50 V rms. These values of  $d\alpha$  and  $d\beta$  for each divider were measured with this circuit using a third divider as a reference. Relative deviations were calculated from these results and listed in the table as "computed"  $d_1(\alpha_2 - \alpha_1)$  and  $d_1(\beta_2 - \beta_1)$  for comparison with those determined by a direct comparison of the two test dividers. The latter are listed as "measured"  $d_1(\alpha_2 - \alpha_1)$  and  $d_1(\beta_2 - \beta_1)$ .

The computed and measured results listed in the table agreed to 0.02 ppm for  $d_1(\alpha_2 - \alpha_1)$  and to 0.1 ppm for  $d_1(\beta_2 - \beta_1)$ .

The voltage ratio is defined by the equation

$$\frac{V_{out}}{V_{in}} = d(1 + \alpha + j\beta)$$

TABLE I  
In-Phase and Quadrature Deviations of Two Inductive Voltage Dividers

Parts per million of input					Parts per million of input			
$d_1$	$d_1\alpha_1$	$d_2\alpha_2$	Computed $d_1(\alpha_2 - \alpha_1)$	Measured $d_1(\alpha_2 - \alpha_1)$	$d_1\beta_1$	$d_2\beta_2$	Computed $d_1(\beta_2 - \beta_1)$	Measured $d_1(\beta_2 - \beta_1)$
0.X	+0.15	+0.16	+0.01	+0.01	-0.4	-0.3	+0.1	0.0
0.9	+1.27	-0.23	-1.50	-1.51	-0.4	-0.3	+0.1	+0.1
0.8	+1.22	-0.31	-1.53	-1.53	-0.3	0.0	+0.3	+0.3
0.7	+2.21	-0.33	-2.54	-2.53	+0.1	0.0	-0.1	-0.2
0.6	+1.90	-0.42	-2.32	-2.32	-0.1	-0.2	-0.1	-0.2
0.5	+1.63	-0.27	-1.90	-1.90	0.0	-0.4	-0.4	-0.4
0.4	+1.17	+0.03	-1.14	-1.14	-0.1	-0.4	-0.3	-0.3
0.3	+0.41	0.00	-0.41	-0.42	-0.4	-0.5	-0.1	-0.2
0.2	+0.34	0.00	-0.34	-0.36	-0.3	-0.6	-0.3	-0.3
0.1	+0.11	+0.04	-0.07	-0.08	-0.3	-0.5	-0.2	-0.2
0.0X	-1.03	+0.21	+1.24	+1.25	-0.2	-0.4	-0.2	-0.2
0.09	-0.96	+0.18	+1.14	+1.16	-0.2	-0.4	-0.2	-0.2
0.08	-0.99	+0.15	+1.14	+1.15	-0.3	-0.4	-0.1	-0.1
0.07	-0.88	+0.12	+1.00	+1.01	-0.2	-0.3	-0.1	-0.1
0.06	-0.96	-0.16	+0.80	+0.81	-0.4	-0.4	0.0	-0.1
0.05	-0.70	-0.01	+0.69	+0.71	-0.3	-0.3	0.0	0.0
0.04	-0.43	-0.21	+0.22	+0.22	-0.2	-0.4	-0.2	-0.2
0.03	-0.09	-0.01	+0.08	+0.08	-0.1	-0.2	-0.1	-0.1
0.02	+0.02	+0.01	-0.01	-0.01	0.0	-0.1	-0.1	-0.1
0.01	-0.01	0.00	+0.01	+0.01	-0.1	-0.1	0.0	0.0
0.00X	-0.01	0.00	+0.01	+0.01	-0.1	-0.1	0.0	0.0
0.009	-0.01	-0.01	0.00	0.00	-0.1	-0.1	0.0	0.0
0.008	+0.01	0.00	-0.01	-0.01	-0.1	-0.1	0.0	0.0
0.007	+0.03	0.00	-0.03	-0.02	-0.1	-0.1	0.0	0.0
0.006	+0.04	+0.01	-0.03	-0.03	-0.1	-0.1	0.0	0.0
0.005	+0.05	+0.01	-0.04	-0.04	-0.1	-0.1	0.0	0.0
0.004	+0.06	+0.01	-0.05	-0.05	-0.1	-0.1	0.0	0.0
0.003	+0.07	+0.01	-0.06	-0.06	-0.1	-0.1	0.0	0.0
0.002	+0.09	+0.02	-0.07	-0.07	-0.1	-0.1	0.0	0.0
0.001	+0.10	+0.02	-0.08	-0.08	-0.1	-0.1	0.0	0.0
0.000X	+0.11	+0.02	-0.09	-0.09	-0.1	-0.1	0.0	0.0

and phase angle by

$$\tan \theta = \frac{\beta}{1 + \alpha} \approx \beta$$

### CONCLUSIONS

A high-accuracy comparator has been developed for measuring the relative deviation in voltage ratio and phase angle of inductive voltage dividers from 1 to 10 KHz. An overall accuracy of better than one part in  $10^7$  is achieved.

The new technique overcomes several inherent limitations of the existing comparison methods and has a broader frequency range. It requires only a few pieces of special equipment, and these are easy to construct. This circuitry appears worthy of general use in standardizing laboratories.

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